Strong Cosmic Censorship and Quantum Fields

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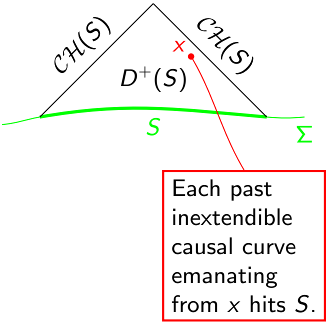
based on arXiv:1912.06047 [with S. Hollands & R.M. Wald]
1st virtual LQP workshop, June 2020

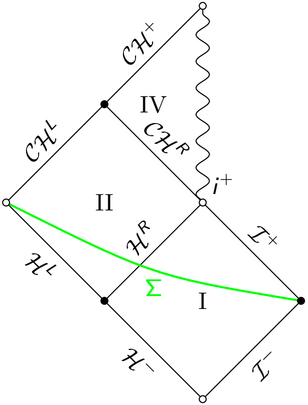
Determinism

- A field ϕ subject to a hyperbolic field equation, e.g. $(\Box - \mu^2)\phi = 0$, is determined by initial data on S within the domain of dependence $D^+(S)$.
- Values beyond the Cauchy horizon $\mathcal{CH}(S)$ not determined.
- The strong cosmic censorship (sCC) conjecture asserts that determinism generically holds in GR, given initial data which is, in a suitable sense, complete (e.g., asymptotically flat).
- Cauchy horizons should be generically singular, so that no observer may cross them.



Challenges to sCC: Reissner-Nordström spacetime

- Reissner-Nordström spacetime (RN) has a regular Cauchy horizon $\mathcal{CH}^R$, beyond which a field ϕ is not determined by its data on a Cauchy surface Σ.



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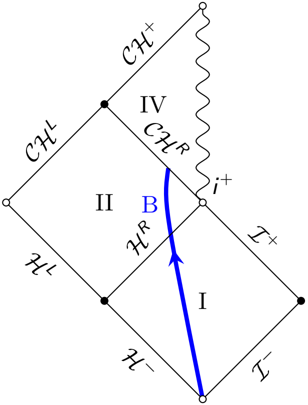

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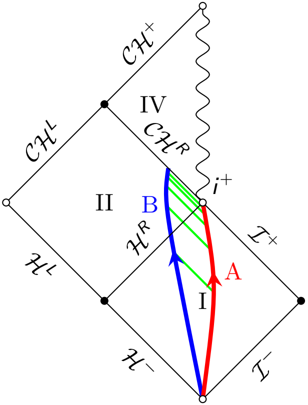
- Alice, who does not enter the black hole, sends periodic signals to Bob. She needs ∞ proper time to reach i^+, so she may send ∞ many of those. As Bob receives them in finite proper time, the frequency diverges as he approaches $\mathcal{CH}^R$.
- For generic perturbations of fields on RN one expects a divergence of the stress tensor and thus the curvature as $\mathcal{CH}^R$ is approached [Penrose 1974].



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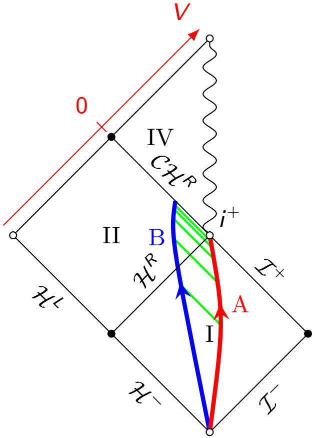
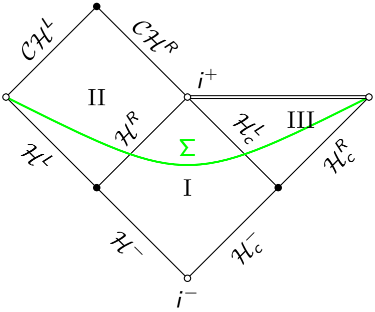
- For generic perturbations of fields on RN one expects a divergence of the stress tensor and thus the curvature as $\mathcal{CH}^R$ is approached [Penrose 1974].
- Christodoulou formulation: sCC holds if generically $\phi \notin H^1_{\text{loc}}$ near $\mathcal{CH}^R$, i.e., divergence at least as

$$T_{VV} \sim V^{-1}.$$



Challenges to sCC: Reissner-Nordström-deSitter spacetime



- With a positive cosmological constant Λ, the blue-shift of the frequency is counteracted by the cosmological expansion, so that [Hintz & Vasy 2017]

$$\phi \in H^{\frac{1}{2}+\beta}_{\mathrm{loc}}, \qquad T_{VV} \sim (-V)^{-2+2\beta}$$

  with

$$\beta = \frac{\alpha}{\kappa_-} = \frac{\text{spectral gap of QNMs}}{\text{surface gravity at } \mathcal{CH}}$$

  on Reissner-Nordström-deSitter (RNdS).
- Near extremal RNdS, scalar fields: $\beta > \frac{1}{2}$ [Cardoso et al 2017], [Dias et al 2018].
- Near extremal RNdS, linearized Einstein-Maxwell: $\beta > 2$ [Dias at al 2018].
- sCC violated on RNdS (but not on Kerr-dS [Dias et al 2018]).

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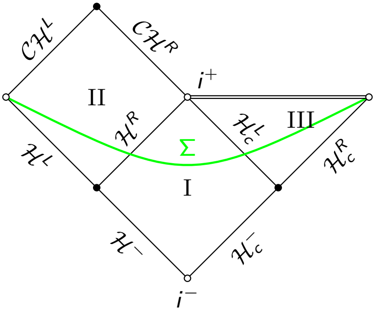

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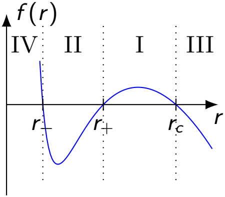
- sCC violated on RNdS (but not on Kerr-dS [Dias et al 2018]).
- We find that on RNdS, in any state Ψ which is Hadamard around Σ,

$$\langle T_{VV} \rangle_\Psi \sim C V^{-2}$$

near $\mathcal{CH}^R$ with C generically non-vanishing and state-independent.
- sCC rescued by quantum effects.

RNdS spacetime



- Metric given by

$$g = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2,$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2,$$

- Roots $0 < r_- < r_+ < r_c$ of f are the Cauchy, event, cosmological horizon.
- The corresponding surface gravities are $\kappa_i = \frac{1}{2}|f'(r_i)|$ for $i \in \{-, +, c\}$.

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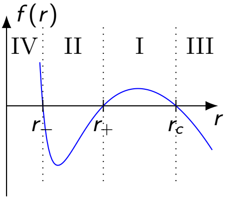

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- Introduce radial null coordinates u, v such that

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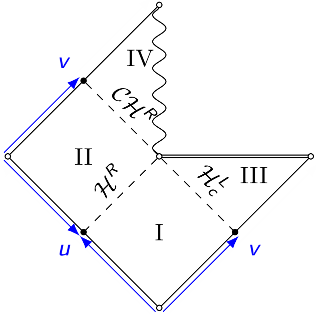

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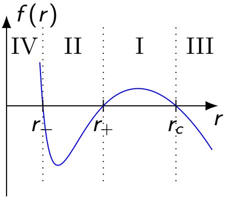

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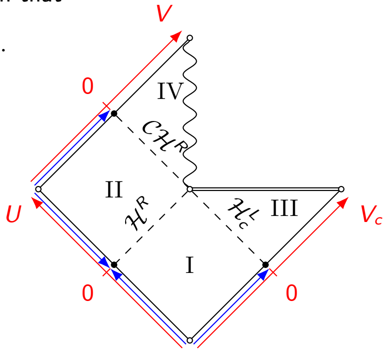
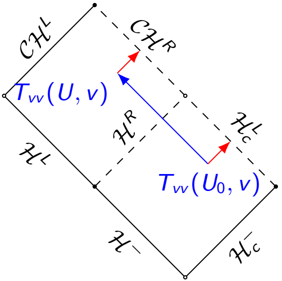
- In Kruskal coordinates U, V, V_c, we can extend the metric analytically over $\mathcal{H}^R$, $\mathcal{CH}^R$, and $\mathcal{H}^L_c$.



The 2d case

- 2d toy model in which the angular directions are suppressed: $g = -f(r)\mathrm{d}u\mathrm{d}v$.
- Classically, stress tensor conserved and traceless, so $\partial_u T_{vv} = 0$, implying that

$$T_{VV}(U, v) = T_{V_c V_c}(U_0, v)\frac{\kappa_c^2}{\kappa_-^2}(-V)^{-2+2\kappa_c/\kappa_-}.$$



- If field is regular at $\mathcal{H}_c^L$, then generically singular at $\mathcal{CH}^R$.
- Exponent dependent on spacetime parameters, coefficient state-dependent.

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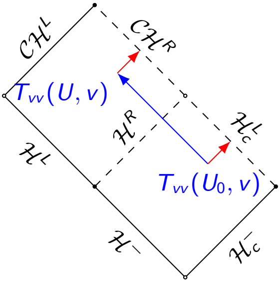

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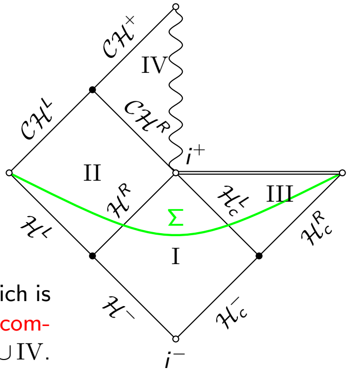
- If field is regular at $\mathcal{H}_c^L$, then generically singular at $\mathcal{CH}^R$.
- Exponent dependent on spacetime parameters, coefficient state-dependent.
- For a quantum field, trace anomaly: $T = aR$.
- Integration of $\partial_u\langle T_{vv}\rangle_\Psi$ now yields, near $\mathcal{CH}^R$ [Birrell & Davies 1978]

$$\langle T_{VV}\rangle_\Psi = \underbrace{\tfrac{a}{2}(\kappa_c^2/\kappa_-^2 - 1)}_{C} V^{-2} + \mathcal{O}((-V)^{-2+2\kappa_c/\kappa_-})$$

- Power law singularity at $\mathcal{CH}^R$, exponent universal, coefficient C dependent on spacetime parameters and state-independent.
- $C \neq 0$ up to special spacetime parameters and both signs possible.

The 4d case



- In 4d, trace anomaly and conservation are not sufficient to integrate the stress tensor: Unknown state-dependent tangential pressures enter [Birrell & Davies 1978].
- We define a stationary Unruh state $\langle\cdot\rangle_{\mathrm{U}}$, which is Hadamard in $\mathrm{I} \cup \mathrm{II} \cup \mathrm{III}$, and a stationary comparison state $\langle\cdot\rangle_{\mathrm{C}}$, which is Hadamard in $\mathrm{II} \cup \mathrm{IV}$.
- To investigate the divergence of the stress tensor near $\mathcal{CH}^R$, we compute

$$\langle T_{VV}\rangle_\Psi = \langle T_{VV}\rangle_\Psi - \langle T_{VV}\rangle_{\mathrm{U}} + \langle T_{VV}\rangle_{\mathrm{U}} - \langle T_{VV}\rangle_{\mathrm{C}} + \langle T_{VV}\rangle_{\mathrm{C}}$$

Can be controlled using results for the classical case. Yields $\sim (-V)^{-2+2\beta}$.

Can be evaluated numerically. Yields $\sim CV^{-2}$.

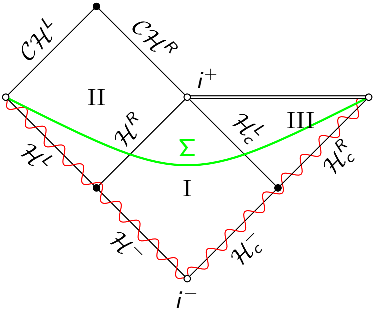
Regular across $\mathcal{CH}^R$

$\langle T_{VV}\rangle_\Psi - \langle T_{VV}\rangle_{\mathrm{U}}$

- $\langle\cdot\rangle_{\mathrm{U}}$ defined as the vacuum w.r.t. to the Unruh modes [Hawking 1975, Unruh 1976]

$$\Psi^{\mathrm{in}}_{k\ell m} \sim Y_{\ell m}(\theta,\phi)e^{-ikV_c} \qquad \text{on } \mathcal{H}_c^- \cap \mathcal{H}_c^R$$
$$\Psi^{\mathrm{up}}_{k\ell m} \sim Y_{\ell m}(\theta,\phi)e^{-ikU} \qquad \text{on } \mathcal{H}^- \cap \mathcal{H}^L$$

- Well-definedness and Hadamard property from propagation of singularities and decay properties on RNdS [Hintz & Vasy 2017] similarly to Schwarzschild case [Dappiaggi, Moretti, Pinamonti 2011].



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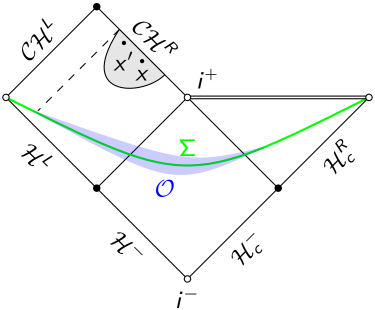


$W(x,x') = \langle \phi(x)\phi(x') \rangle_\Psi - \langle \phi(x)\phi(x') \rangle_{\mathrm{U}}$ smooth in $\mathrm{I} \cup \mathrm{II} \cup \mathrm{III}$.

[Verch 1994]

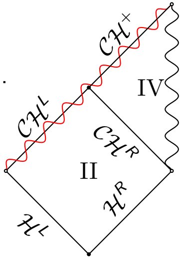
$W(x,x') = \sum_j \pm \bar{\psi}_j(x)\psi_j(x')$
$(\Box - \mu^2)\psi_j = b_j \in C_0^\infty(\mathcal{O})$
$\sum_j \|b_j\|^2_{C^m} < \infty \ \forall m$

Decay estimates [Hintz & Vasy 2017] and Sobolev embedding thms:
$\|t_{\mu\nu}(-,y)\|_{L^p(\mathbb{R}_-)} \lesssim \sum_j \|b_j\|^2_{C^m}$
for $1/p > 2 - 2\beta$ with $\beta > \frac{1}{2}$.

$$\boxed{\langle T_{VV} \rangle_\Psi - \langle T_{VV} \rangle_{\mathrm{U}} \lesssim (-V)^{-2+2\beta}.}$$

$\langle T_{vv} \rangle_{\mathrm{U}} - \langle T_{vv} \rangle_{\mathrm{C}}$

- $\langle \cdot \rangle_{\mathrm{C}}$ as the vacuum state w.r.t. Unruh modes on $\mathcal{CH}^L \cup \mathcal{CH}^+$.
- Is Hadamard in $\mathrm{II} \cup \mathrm{IV}$.



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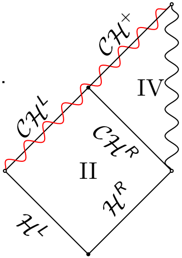
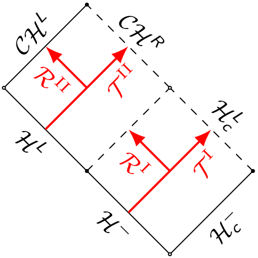
- Compute $\tilde{C} = \langle T_{vv} \rangle_{\mathrm{U}} - \langle T_{vv} \rangle_{\mathrm{C}}$ on $\mathcal{CH}^L$:

$$\tilde{C} \sim \sum_{\ell} (2\ell + 1) \int_0^{\infty} \mathrm{d}\omega \; \omega n_\ell(\omega).$$



- The "density of states" $n_\ell(\omega)$ expressable in terms of transmission and reflection coefficients $\mathcal{T}_{\omega\ell}$, $\mathcal{R}_{\omega\ell}$ for Boulware modes $e^{-i\omega u}$. Must be computed numerically.
- By stationarity, we have the same value on $\mathcal{CH}^R$, so

$$\langle T_{VV} \rangle_{\mathrm{U}} - \langle T_{VV} \rangle_{\mathrm{C}} \sim \tilde{C} \kappa_-^{-2} V^{-2}.$$



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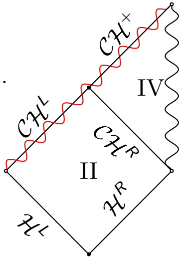

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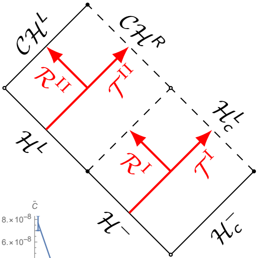
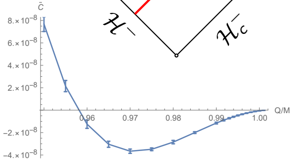
$$\langle T_{VV}\rangle_{\mathrm{U}} - \langle T_{VV}\rangle_{\mathrm{C}} \sim \tilde{C}\kappa_-^{-2}V^{-2}.$$

- Generically $\tilde{C} \neq 0$, both signs possible.
- Compatible with results on RN [Zilberman et al 2019].





[Hollands, Klein, Z. 2020]

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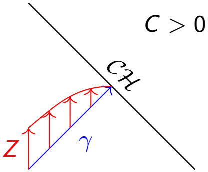
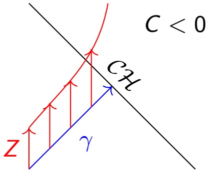
- Infinite crushing ($C > 0$) or stretching ($C < 0$) of observer.





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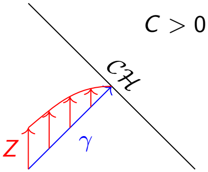

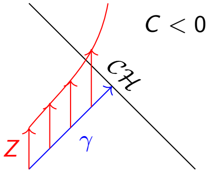

THANK YOU FOR YOUR ATTENTION!